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Title: Numerical rank of kernel functions

Abstract: We study the rank of sub-matrices arising out of kernel functions, $F(\mathbf{x}, \mathbf{y}) : \mathbb{R}^d \times \mathbb{R}^d \mapsto \mathbb{R}$, where $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ with $F(\mathbf{x}, \mathbf{y})$ is smooth everywhere except along the line $\mathbf{y} = \mathbf{x}$. Such kernel functions are frequently encountered in a wide range of applications such as N-body problems, Green's functions, integral equations, geostatistics, kriging, Gaussian processes, radial basis function interpolation, etc. One of the challenges in dealing with these kernel functions is that the corresponding matrix associated with these kernels is large and dense; thereby, the computational cost of matrix operations is high. We prove new theorems bounding the numerical rank of sub-matrices arising out of these kernel functions. Under reasonably mild assumptions, we prove that certain sub-matrices are rank-deficient in finite precision. The rank of these sub-matrices depends on the dimension of the ambient space and also on the type of interaction between the hyper-cubes containing the corresponding set of particles. This rank structure can be leveraged to reduce the computational cost of certain matrix operations such as matrix-vector products, solving linear systems, etc. We also present numerical results on the growth of rank of certain sub-matrices in 1D, 2D, 3D and 4D, which, not surprisingly, agrees with the theoretical results.