Mathematical modeling of the lubrication process inside a human knee joint

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Viscous flow through porous media has potential applications in fields like oil recovery, drug transport through biological tissues, insulations, engineering salvaging process etc. The major parameters that influence various flow quantities like velocity, pressure and stress are porosity and permeability. In particular, permeability is more responsible due to its isotropic or anisotropic nature. If the pores are distributed in an arbitrary manner it causes anisotropy. For granular porous media the distortion of grains in an arbitrary manner causes anisotropy. For the case of fibrous media, the arbitrary orientation of fibres in different direction forms an anisotropic network inside the porous medium. We introduce the concept of anisotropy and the corresponding mathematical structure. We show the variations of anisotropy versus isotropy with some examples. We then discuss a theoretical model of squeeze-film in the presence of a porous bed. This model consists of a flat impermeable bearing that is being squeezed towards a porous interface. The gap between the porous bed and the bearing is assumed to be filled with a Newtonian fluid. This is an approximation to the lubrication process inside a human knee joint. We use Navier-Stokes equation in the fluid region and Darcy equation in the fluid filled porous region. Lubrication approximation is used to derive the corresponding evolution equation for the film thickness. We use Beavers-Joseph condition and Bars-Worster condition at the liquid porous interface and present a detailed analysis on the corresponding impact. We assume that the porous bed is anisotropic in nature with permeabilities K_2 and K_1 along the principal axes. Accordingly, the anisotropic angle ϕ is taken as the angle between the horizontal direction and principal axis with permeability K_2. We show that the anisotropic permeability ratio and the anisotropic angle make a significant influence on the contact time, flux, velocity, etc. Contact time to meet the porous bed when the bearing approach under a constant prescribed load is estimated. We present some important findings (relevant to knee joint) based on the anisotropic properties of the human cartilage. For a prescribed constant load we have estimated the time duration a healthy human knee remains fluid lubricated.