Formal Language Theory

Problem Sheet 1

1. Find Regular Grammars for the following languages on $\{a, b\}$
   (a) $L = \{w \ : \ n_a(w)$ and $n_b(w)$ are both even$\}$.
   (b) $L = \{w \ : \ (n_a(w) - n_b(w)) \bmod 3 = 1\}$.
   (c) $L = \{w \ : \ (n_a(w) - n_b(w)) \bmod 3 \neq 0\}$.

2. Find a regular grammar that generates the set of all Pascal real numbers.

3. Find the minimal DFA for the following languages
   (a) $L = \{a^n b^m : n \geq 2, m \geq 1\}$.
   (b) $L = \{a^n b : n \geq 0\} \cup \{b^n a : n \geq 1\}$.
   (c) $L = \{a^n : n \geq 0, n \neq 3\}$.

4. Find regular expression for the set $\{a^n b^m : (n + m)$ is even$\}$.

5. Give a regular expression for the following languages:
   (a) $L = \{a^n b^m : n \geq 4, m \leq 3\}$.
   (b) $L = \{a^n b^m : n \geq 1, m \geq 1, nm \geq 3\}$.
   (c) $L = \{ab^n w : n \geq 3, w \in \{a, b\}^+\}$.
   (d) $L = \{w \in \{0, 1\}^* : w$ has exactly one pair of consecutive zeros$\}$.
   (e) $L = \{w \in \{0, 1\}^+ : w$ ends with $01\}$.
   (f) $L = \{w \in \{0, 1\}^+ : |w|_0 =$ even$\}$.

6. Prove the following:
   (a) $(r_1^*)^* \equiv r_1^*$.
   (b) $r_1^*(r_1 + r_2)^* \equiv (r_1 + r_2)^*$.
   (c) $(r_1 + r_2)^* \equiv (r_1^* r_2^*)^*$.
   for all regular expression r_1 andr_2. Here $\equiv$ stands for equivalence in the sense of the language generated.

7. Find an NFA that accepts the language $L(aa^*(a + b))$.

8. Find DFA that accepts the following languages:
   (a) $L(aa^* + aba^*b^*)$.
   (b) $L(ab(a + ab)^* + (a + aa))$.
   (c) $L((abab)^* + (aaa^* + b)^*)$.
   (d) $L(((aa^*)^*b)^*)$.

9. Construct a DFA that accepts the language generated by the grammar

$$S \rightarrow abA$$

$$A \rightarrow baB$$

$$B \rightarrow aA/bb$$

10. Construct right and left linear grammars for the following language:

$$L = \{a^n b^m : n \geq 2, m \geq 3\}$$

11. Construct a right linear grammar for the following language:

$$L((aab^*ab)^*)$$