# MA6230 GRAPH THEORY

Basics: subgraphs, isomorphism, automorphism group, matrices associated with graphs, degrees, walks, connected graphs, shortest path algorithms.

Connectivity: Connectivity and Mengers theorem; Structure of 2-connected and 3-connected graphs, Maders theorem.

Matchings: Berge's theorem, Hall's theorem, Tutte's perfect matching theorem, k-matchings (reduction to perfect matching problem), job-assignment-problem.

Ramsey Theory: Pigeonhole Principle, Ramsey Theorem, Ramsey Numbers.

Eulerian and Hamiltonian graphs: characterization of Euler graphs, necessary/ sufficient conditions for the existence of Hamilton cycles, Fleury's algorithm for eulerian trails, Chinese-postman-problem (complete algorithmic solution), approximate solutions of traveling salesman problem.

Planar Graphs: Euler's formula, Dual graphs, Characerization of planar graphs, planarity testing. Coloring: Brooks' Theorem, Graphs with large chromatic number, Turan's theorem.

References :

Books:

1. D.B. West, Introduction to graph theory, PHI Learning, New Delhi, 2004.

References:

1. J.A. Bondy and U.S.R. Murty, Graph Theory and Applications, GTM Vol. No. 244, Springer 2008.

2. R. Diestel, Graph Theory, Springer, 2006.

## MA 5013 Applied Regression Analysis

Unit I: Simple linear regression, multiple linear regression, model adequacy checking, transformations and weighting to correct model inadequacies.

Unit II: Polynomial regression models, orthogonal polynomials. dummy variables, variable selection and model building, multicollinearity.

Unit III: Nonlinear regression. Generalized linear models, autocorrelation, measurement errors, calibration problem, bootstrapping.

References :

Text Books:

1. 1. Montgomery, D. C., Peck, E. A., and Vining, G. (2012), Introduction to Linear Regression Analysis (5th ed.), Hoboken, NJ: Wiley.

References:

1. Draper, N. R., and Smith, H. (2003), Applied Regression Analysis, New York: Wiley.

2. Sen, A. A. and Srivastava, M. (1990). Regression Analysis Theory, Methods & Applications, Springer-Verlag, Berlin.

3. Bowerman, B. L. and O'Connell, R. T. (1990). Linear Statistical Models: An Applied Approach, PWS-KENT Pub., Boston.

### MA 5014 Applied Stochastic Processes

Discrete-Time Markov Models: Discrete-Time Markov Chains, Transient Distributions, Occupancy Times, Limiting Behavior, First-Passage Times. **Poisson Processes:** Poisson Processes, Superposition of Poisson Processes, Thinning of a Poisson Process, Compound Poisson Processes. Continuous-Time Markov Models: Continuous-Time Markov Chains, Transient Analysis: Uniformization, Occupancy Times, Limiting Behavior, First-Passage Times. Generalized Markov Models: Renewal Processes, Cumulative Processes, Semi-Markov Processes. **Queueing Models:** Queueing Systems, Single-Station Queues, Birth and Death Queues. **Brownian Motion:** Standard Brownian Motion, Brownian Motion, First-Passage Times, Martingales and Semimartingales, Black Scholes Formula.

References :

Text Books:

1. V. G. Kulkarni, Introduction to modeling and analysis of stochastic systems, second edition, Springer, 2011.

References:

1. S. M. Ross, Stochastic processes, second edition, Wiley, 1996.

2. S. Karlin and H. M. Taylor, A first course in stochastic processes, second edition, Academic Press, 1975.

3. S. M. Ross, Introduction to Probability Models, tenth edition, Academic Press, 2009.

## MA5016 ERGODIC THEORY Course Content:

Basic Concepts "Ergodicity, Freeness, Poincare Recurrence, Almost Periodicity, Weak mixing, Mixing; Structure Theorems: Furstenberg-Zimmer structure theorem, Hilbert modules, Extensions; Amenablity: Ameanble groups, Ergodic Theorems, Quasi Tilings, Rokhlin Lemma. Borel Equivalence Relations: Connes-Feldman-Wiess Theorem.

References :

Text Books:

1. Peter Walters -- An Introduction to Ergodic Theory, Springer, ISBN 978-0-387-95152-2

2. Eli Glasner " Ergodic Theory via Joinings, AMS, ISBN-10: 0-8218-3372-3

3. Ergodic Theory � " Hanfeng Li and David Kerr, online book.

Reference Books: Online Lecture Notes --

1. Jesse Peterson Vanderbilt University " Ergodic Theory

2. Blog/Course Notes of Terrence Tao " Ergodic Theory

### **MA5017 REPRESENTATION THEORY**

Representation of finite groups: Definitions, Basic examples, sub representations, irreduciblerepresentations, tensor product of two representation, character of representation, Schur's lemma, orthogonality of characters, regula representation, induced representation, representation of compact groups, various examples.

References :

Text Books: Introduction to representation theory- J.P. Serre, Springer GTM Reference Books: Representation Theory- Fulton and Harris, Springer GTM Prerequisite: ma5310-linear algebra,ma5320-algebra-1

# MA5140 INTRODUCTION TO ALGEBRAIC TOPOLOGY

Some Basic Topological Notions: Homotopy, Convexity, Contractibility, Paths and Path Connectedness ; Fundamental group and Covering Spaces: The Fundamental Group of the circle, Induced Homomorphisms Free Products of Groups, Seifert Van Kampen's Theorem, Covering Spaces, Lifting Properties, The Classification of Covering Spaces, Deck Transformations and Group Actions.

References :

Books:

 Allen Hatcher, Algebraic Topology, Cambridge University Press, New York 2002.
 W. S. Massey, A Basic Course in Algebraic Topology, Graduate Texts in Mathematics 127, Springer Verlag, New York 1991.

## MA5220 CONTINUUM MECHANICS

Algebra of Cartesian Tensors, Index notation, Isotropic tensors, Invariants of a tensor, Polar decomposition theorem.

Course contents: Continuum Hypothesis, Kinematics of deformation, Compatibility conditions, Balance principles, Euler's laws of motion, Stress-Laws of Thermodynamics, Energy Balance, Constitutive equations of fluids and elastic materials, principles of material objectivity, Applications (simple problems of fluid dynamics and elasticity)

References :

Books:

D.S. Chandrasekhraiah and Loknath Debnath, Continum Mechanics, Academic Press (1995)

## MA 5311 Linear Systems Theory

Introduction: Physical systems, models and representations.

Linear system representation: Definitions, state transition map, response map, impulse response matrix, adjoint equations, linear quadratic optimization, applications.

Linear time invariant systems: General properties, minimal polynimial, decomposition theorem, the linear map  $X \rightarrow AX+XB$ .

Stability: Input-output stability, state related stability concepts and applications.

Controllability and Observability: Controllability and observability of dynamical systems, Controllability of the pair (A(.),B(.)), observability of the pair (C(.),A(.)), duality, Kalman decomposition theorem, minimal realization, controllable canonical form.

Linear state feedback and estimation: Linear state feedback, linear output injection and state estimation, feedback of the estimated state, infinite time horizon linear quadratic optimization.

### References :

Text Books:

1. L.A. Zadeh and C.A. Desoer, Linear System Theory: The state space approach, Dover, 2008. Reference:

1. P.J. Antsaklis and A. N. Michel, Linear Systems, Birkhauser, 2005.

- 2. Chi-Tsong CHen, Linear System Theory and Design, Oxford Univ Press, 3rd edition, 1998.
- 3. W. J. Rugh, Linear System Theory, Prentice Hall, 2nd edition, 1995.
- 4. T. Kailath, Linear Systems, Prentice Hall, 1980.

## MA 5312 Stochastic Differential Equations

Introduction: Stochastic analogs of classical differential equations.

Mathematica; preliminaries: Probability space, random variable, stochastic process, Brownian motion.

Ito Integral: Definition, Properties, extensions.

Ito formula and Martingale representation Theorem: One-dimensional Ito formula, Multidimensional Ito formula, Martingle representation Theorem.

Stochastic differential equations: Examples and some solution methods, Existence an Uniqueness result, weak and strong solutions.

Applications: Boundary valur problems, filtering, optimal stopping, stochastic control, mathematical finance.

References :

Text Books:

1. B. K. Oksendal, Stochastic Differential Equations: An Introduction with Applications, 6th edition, Apringer, 2010.

Reference:

1. I. Karatzas and S. E. Shreve, Brownian Motion and Stochastic Calxulus, SPringer, 1991.

2. P. Protter, Stochastic Integration and Differential Equations, Springer, 2nd edition. 2010.

3. I. Karatzas and S.E. Shreve, Methods of Mathematical Finance, Springer, 2010.

4. S. Watanabe and N. Ikeda, Stochastic Differential Equations and Diffusion Processes, North-Holland, 1981.

### MA 5313 Introduction to Mathematical Statistics

Unit I: Distributions of Random Variables: The probability set function; random variables; probability density function; distribution function; certain probability models; mathematical expectation; Chebyshev () s inequality; conditional probability; marginal and conditional distributions; correlation coefficient; stochastic independence.

Unit II: Special Discrete and Continuous Distributions and the Distributions of Functions of Random Variables. Sampling Theory; Transformation of variables: Discrete and Continuous; Distribution of order statistics; moment generating function technique; Distribution of  $\bar{X}$  and S2 and expectations.

Unit III: Limiting Distributions and Estimation: Limiting distributions; stochastic convergence; limiting moment-generating functions; central limit theorem; point estimation, measures of quality of estimators; Confidence intervals: mean, differences of means; variances; Bayesian estimators.

Unit IV: Statistical Hypotheses and other Statistical Tests: Testing of hypotheses, Neyman-Pearson lemma, tests for one sample and two sample problems; chi-squared tests; analysis of variance; regression problem; test for stochastic independence.

References :

Text Books:

1. Hogg R. V. and Craig, A. T., Introduction to Mathematical Statistics, Macmillan Publishing Co., Fourth Edition, (1989).

References:

1. G. Casella and R. L. Berger. Statistical Inference. Duxbury Press, (2001).

2. V. K. Rohtagi and A. K. Md. E. Saleh. An Introduction to Probability and Statistics. Wiley Eastern, Second Edition (2002).

# MA 5430 ALGEBRA II: RING THEORY AND FIELD THEORY

Module Theory: Right and Left modules, Examples over Z, polynomial rings, Quotient modules, module homomorphism & homomorphism theorems .

Ring Theory: Structure Theorem of modules over PID, a polynomial ring over a UFD is again a UFD, basic properties of Noetherian rings and modules, basic properties of localisation .

Field Theory: Classical Straightedge and Compass construction and examples, normal, separable and Galois extensions.

Basic Non-Commutative Algebra: Linear Maps and Modules over non-commutative rings, Simple and Semi-simple modules and rings, Schur's Lemma, Jacobson's Density Theorem, Burnside Lemma and Wedderbern Theorem, Matrix rings over division algebras are simple.

## References :

Text Books:

1. D. S. Dummit and R. M. Foote: Abstract Algebra, 2nd Edition, John-Wiley, 1999.

2. S. Lang: Algebra 3rd Edition, Addison-Wesley, 1999.

Reference:

1. J.A. Gallian: Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.

2. M. Artin: Algebra, Prentice Hall inc 1994.

3. I.N. Herstein: Topics in Algebra, John-Wiley, 1995.

4. T. A. Hungerford: Algebra, Graduate Texts in Mathematics, Vol. 73, Springer-Verlag, 1980.

### MA5440 Combinatorics and Number Theory

Combinatorics: Numbers and counting, partitions and permutations, principle of inclusion and exclusion, pigeon hole principle, recurrence relations, generating Functions. Number Theory: Primes, divisibility and the fundamental theorem of arithmetic, prime number theorem, Euclidean algorithm, congruences, ring of integers mod n, chinese remainder theorem, arithmetic functions, Fermat's last theorem, Mobius inversion formula, quadratic residues, quadratic reciprocity law, binary quadratic forms, continued fractions, Pell's equation, Diophantine equations.

References :

Texts books:

1. J. L. Mott, A. Kandel, and T. P. Baker, Discrete Mathematics for Computer Scientists and Mathematicians, PHI Learning, 2003.

2. I. Niven, H.S. Zuckerman, and H.L. Montgomery. An Introduction to the Theory of Numbers, Wiley, 1991.

References:

1. T. Koshy, Discrete Mathematics with Applications, Elsevier, 2004.

2. K.F. Ireland and M.I. Rosen, A Classical Introduction to Modern Number Theory, Springer, 1990.

# MA 5460 TRANSFORM TECHNIQUES

Riemann - Lebesgue Lemma, localization lemma, Fourier integral theorem, Fourier Transform, Inversion, Convolution and Parseval's Theorem, Applications to Partial Differential Equations. Laplace transform :Definition, properties, Complex inversion, Applications to initial and boundary value problems.

Z - Transform and Difference equations

References :

Text Books:

1. I.N. Sneddon, The Use of Integral Transforms, Tata Mc-Graw Hill (1974)

2. J.L. Schiff, The Laplace Transform, Springer (1999)

3. Michael Frazier, An Introduction to wavelets through Linear Algebra, Springer, 1999 (Chapter 2)

4. A.V. Oppenheim & R.W. Schafer, Digital Signal Processing, Prentice-Hall, 1975. References:

1.M.R. Spiegel, Laplace Transforms (Schaum's Series), McGraw-Hill, 1965.

2.M.R. Spiegel, Advanced Mathematics for Engineers and Scientists, (Schaum's Series), McGraw Hill, 1983.

3.Jeffery M. Cooper, Introduction to PDE with MATLAB (Chapter 6) Birkhauser, 1997.

### **MA 5490 FLUID DYNAMICS**

Kinematics of Fluid flow, Laws of fluid motion, Inviscid incompressible flows, two and threedimensional motions, Airfoil theory, inviscid compressible flows;

Viscous incompressible flows, Navier-Stokes equations of motion and some exact solutions; Flows at small Reynolds numbers; Boundary layer theory.

References :

Text Books:

1. M.E.O Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics, John Wiley and Sons, New York, 1986.

2. M.E.O Neioll and FD. Chorlton, Viscous and compressible Fluid Dynamics, John Wiley and Sons, New York, 1989.

References:

1. F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers, 1998.

2. A.J. Chorin and J.E. Marsden, A Mathematical Introduction to Fluid mechanics, Springer-Verlag, 1999.

3. G-Emanuel, Analytical Fluid Dynamics, CRC Press, 2001.

4. P.K. Kundu and I.M. Cohen, Fluid Mechanics (3rd edition) Elsevier Science & Technology, 2002.

# MA5750 APPLIED STATISTICS

Review of probability distributions and statistical inference, Design of experiments, Singlefactor, Randomized Blocks, Lotin Square. Regression, linear, multiple, curvilinear, Nonparametric texts, sign, signed-rank, randomness and other nonparametric tests. Statistical Quality control, Control charts for measurements and for attributes, Tolerance limits, Acceptance Sampling. Reliability and life testing.

References :

## **MA5850** Operations Research

Linear Optimization: Formulation and Geometrical Ideas of Linear Programming Problems, Simplex Method, Revised Simplex Method, Duality, Sensitivity Analysis, Transportation and Assignment Problems, Introduction to Interior-Point Methods. (Ellipsoid Method, Karmarkar's Method).

Unconstrained optimization of functions of several variables: Basic theory, Classical techniques and numerical methods for unconstrained optimization (Gradient methods, Newton's method, Conjugate Direction methods, and Quasi-Newton methods).

Constrained nonlinear optimization of functions of several variables: Method of Lagrange multipliers, Kuhn-Tucker theory, Convex optimization, Quadratic optimization, Numerical methods for constrained optimization, Dynamic programming.

References :

Texts books:

1. E.K.P. Chong, and S.H. Zak: An Introduction to Optimization, 3rd Edn., Wiley Interscience, 2008.

References:

1. D. G. Luenberger, Linear and Nonlinear Programming, 2nd Edn., Kluwer, 2003.

2. N. S. Kambo, Mathematical Programming Techniques, East West Press, 1997.

3. M. S. Bazarra, H.D. Sherali, and C. M. Shetty, Nonlinear Programming: Theory and

Algorithms, 2nd Edn., John Wiley, 1993. (also available as WSE (2004) edition).

4. D.P. Bertsekas, Nonlinear Programming, 2nd Edn., Athena Scientific, 1999.

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# MA 5950 MATHEMATICAL FINANCE

Introduction to Stochastic Processes, Poisson Process, Brownian Motion, Martingales Present Value Analysis, Interest rate analysis, Market Model Specification problems. Arbitrage Theorem, Multi-period binomial Model, Block - Scholes formula. Valuing investments by expected utility, Portfolio selection problem, Capital Assets Pricing model, Rates of return, Single period and geometric Brownian motion, Mean-variance analysis of risk - neutra-lpriced call options, Autoregressive models and mean regression, Other pricing options and applications.

# References :

1. S.M.Ross, "An Introduction to Mathematiacl Finance", Cambridge University Press, 1999

 A.J.Prakash, R.M.Bear, K.Dandapani, G.L.G.T.E.Pactwa and A.M.Parchigari, "The return Generating Models in Global Finance", Pergamon Press, 1998
 S.M.Ross, "Applied Probability models with Optimization Applications", Holdenday, 1980.

# MA5890 NUMERICAL LINEAR ALGEBRA

## **Description:**

The course will focus on designing algorithms for matrix computations, analysing these algorithms (in terms of complexity, communication costs, stability, performance in finite precision and exact arithmetic), implementation on different computer architectures.

## **CourseContent:**

Floating point arithmetic (1 lecture) Stability of algorithms (2 lectures) conditioning of a problem (2 lectures) perturbation analysis (2 lectures) algorithmic complexity (1 lecture) Matrix decomposition including LU, Cholesky, QR, SVD, etc. (12 lectures) Iterative techniques mainly focussing on Krylov subspace methods including Lanczos, Arnoldi, Conjugate Gradient, GMRES, etc. (12 lectures) Preconditioning (2 lectures) structured matrix computations (4 lectures) designing matrix algorithms on modern computer architectures (3 lectures)

References :

TextBooks:
1. James W. Demmel, Applied Numerical Linear Algebra, Publisher : Society for Industrial and Applied Mathematics, Year : 1997
2. N. Trefethen & David Bau III, Numerical Linear Algebra, Publisher : Society for Industrial and Applied Mathematics, Year : 1997

ReferenceBooks: 1. Biswa Nath Datta, Numerical Linear Algebra and applications, 2nd Edition, Publisher : Society for Industrial and Applied Mathematics, Year : 2010

Prerequisite: Linear Algebra at undergraduate level

### MA6001 Introduction to coding Theory

Introduction to linear codes and error correcting codes. Encoding and decoding of a linear code, dual codes. Hamming codes and perfect codes. Cyclic codes. Codes with Latin Squares, Introduction to BCH codes. Weight ennumerators and MDS codes. The main linear coding theory problems and conclusions.

References :

Books:

Raymond Hill, A first course in Coding Theory, Clarandon Press, Oxford (1986).
 J. H. Van Lint, Introduction to Coding Theory, Springer (1998).

## MA6002 APPROMIMATION THEORY

Approximation of Periodic Functions, Approximation by Algebraic Polynomials, Convergence of Bernstein Polynomials, Korovkin's Theorem, The Stone- Weierstrass Theorem Approximation in Normed Linear Spaces, Linear Chebyshev Approximation of Vector - valued Functions, Uniqueness Results and Haar Subspaces.

Strong Uniqueness and Continuity of Metric Projection, Upper Semicontinuity of Metric Projections, Lower Continuity, Continuous Selections and Lipschitz Continuity of Metric Projections.

References :

Books:

1 N. Mhaskar and Devidas V.Pai , Fundamentals of Approximation Theory, Revised Edition Narosa , 2007.

### MA6003 THEORY OF WAVELETS

Review of Fourier series, Fourier transform of square-integrable functions, Orthonormal bases generated by a single function: the Balian-Low theorem, The Gabor transform, Short-time Fourier transform and the Uncertainty Principle, The integral wavelet transform, Dyadic wavelets and inversions, Frames, Wavelet Series, Multiresolution analysis, Construction of wavelets from a multiresolution analysis, The construction of compactly supported wavelets, Applications of wavelets to signal processing.

References :

Books:

- 1. C.K. Chui, An introduction to wavelets, Academic Press.
- 2. I. Daubechis, Ten Lectures on wavelets, SIAM.
- 3. S. Mallat, A wavelet tour of signal processing, Elsevier.

## MA6004 FRACTALS

Classical Fractals, Self-similarity, Metric Spaces, Equivalent Spaces, The Space of Fractals, Transformation on Metric Spaces, Contraction Mapping and Construction of fractals from IFS. Fractal Dimension, Hausdorff measure and dimension, Fractal Interpolation Functions, Hidden Variable FIF, Fractal Splines, Fractal Surfaces, Measures on Fractals, Complex Dynamics, Mandelbrot Set, Julia Set and Fatou Set.

References :

Books:

1. M.F. Barnsley, Fractals Everywhere, Academic Press.

2. K. Falconer, Fractal Geometry (Mathematical Foundations and Applications), John Wiley  $\$  Sons.

3. Robert Devaney, A First Course in Dynamical Systems, Addison-Wesley

# MA6005 APPLIED LINEAR ALGEBRA

Inner Product Spaces: Normal, Unitary and Self-adjoint operators, Finite Dimensional Spectral theorem for normal operators. Quadratic forms, difference equations.

Orthogonal reduction, Discrete Fourier Transform, complementary subspaces, range-null space decomposition, orthogonal decomposition, singular-value decomposition, orthogonal projection, least squares solutions.

Perron Frobenius Theory: Positive matrices, Nonnegative matrices, Stochastic matrices and applications to Markov chains.

References :

Books:1. K. Hoffman and R. Kunze, Linear Algebra, Prentice-Hall, Second Edition,2008.2. C. D. Meyer, Matrix Analysis and Applied Linear Algebra, SIAM, 2001.

### MA6006 APPLIED INTEGRAL EQUATIONS

Classification of Integral Equations, various examples, Abel's problem, 2nd order ordinary differential equations and Integral Equations, Initial and boundary value problems, singular boundary value problems.

Integral Equations of second kind: degenerate kernels, Neumann series.

Compact self-adjoint operators: Structure theorem, spectrum, Applications to integral equations, positive operators and Integral equations arising from Sturm-Liouville theory.

Approximate methods for eigenvalues and eigenvectors of self-adjoint operators, Approximation of integral equations based on variational principles.

Singular integral equations: introduction, solution methods, applications.

References :

Books:

David Porter & David S.G. Stirling, Integral Equations: A Practical Treatment, from Spectral Theory to Applications, Cambridge texts in Applied Mathematics, 1990.

#### MA6007 NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Review of first order equations and characteristics

Weak solutions to hyperbolic equations- discontinuous solutions, shock formation, a formal approach to weak solutions, asymptotic behaviour of shocks

Diffusion Processes-Similarity methods, Fisher's equation, Burgers' equation, asymptotic solutions to Burgers' equations

Reaction diffusion equations-traveling wave solutions, existence of solutions, maximum principles and comparison theorem, asymptotic behaviour

Elliptic equations-Basic results for elliptic operators, eigenvalue problems, stability and bifurcation

Hyperbolic system

References :

Books:

J David Logan, An introduction to nonlinear partial differential equations, John Wiley and Sons, Inc., 1994

### MA6050 DYNAMICAL SYSTEMS

One -dimensional flows: fixed points and stability, linear stability analysis, Saddle- node bifurcation, Transcritical bifurcation, Pitchfork bifurcation, Flows on the circle Two -dimensional flows: Linear systems, nonlinear autonomous systems, phase portraits, Fixed points and linearization, conservative systems, index theory, limit cycles, Poincare Bendixson theorem, Lienard systems, Saddle-node, Transcritical and Pitchfork bifurcations, Hopf bifurcation Chaos

Discrete dynamical systems

### References :

Books:

(1) Steven H Strogatz, Nonlinear dynamics and Chaos, Perseus books publishing, 1994
(2) James T Sandefur, Discrete dynamical systems Theory and applications, Clarendon press, 1990

# MA6060 NONLINEAR CONSERVATION LAWS

Integral and differential forms of conservation laws, Burgers equation, Shock formation, Weak solutions, Entropy conditions, Riemann problem, scalar and non-linear vector examples, Euler equations of an ideal gas, Courant Fredrick Levy conditions, Upwind methods, Conservative schemes, Godunov methods, Riemann solvers.

### References :

Books: R.J. Leveque, Numerical methods for conservation Laws, Bikhauser Verlag, 1992.

## MA6080 FOURIER ANALYSIS

Fourier Series: Definition, Examples, Uniqueness of Fourier series, Convolution, Cesaro summability and Abel summability of Fourier series, Mean square convergence of Fourier series, A continuous function with divergent Fourier series.

Some applications of Fourier series: The isoperimetric inequality, Weyl's equidistribution theorem. Fourier transform: The Schwartz space Fourier transform on the real line and basic properties, Approximate identity using Gaussian kernel, Solution of heat equation, Fourier inversion formula, L2- theory, The class of test functions, Distributions, Convergence, differentiation and convolution of distributions, Tempered distributions, Fourier transform of a tempered distribution.

Some basic theorems of Fourier Analysis: } Poisson summation formula, Heisenberg uncertainty principle, Hardy's theorem, Paley-Wiener theorem, Wiener's theorem, Wiener-Tauberian theorem.

References :

### MA6090 Sobolev Spaces and applications to PDE

Distribution Theory: Test functions and distributions, Convolution of Distributions, Tempered Distributions.

Sobolev Spaces: Definition and basic properties, Extension Theorems, Imbedding theorems, Compactness theorems, Trace theory.

Weak Solutions of Elliptic boundary value problems: Some abstract Variaitional problems, Examples of Elliptic BVPs, Existence and Regularity of weak solutions, Maximum principle, Eigenvalue problems.

References :

Text:

1. S. Kesavan, Topics in Functional Analysis and Applications, New Age International Publishers, 2015.

2. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.

**REFERENCES:** 

1. R.A Adams and J. F. Fournier, Sobolev Spaces, Academic Press, 2003.

2. W. Rudin, Functional Analysis, Tata McGraw-Hill, 2006.

3. R. S. Strichartz, A guide to Distribution Theory and Fourier Transforms, World Scientific, 2008.

4. M. Renardy and R. C. Rogers, An Introduction to Partial Differential Equations, Springer, 2004.

## MA6110 TOPICS IN ADVANCED ANALYSIS

Review of general measure and integral; Positive Borel measures; Riesz representation theorem; Luzin's theorem; Vitali Caratheodory theorem. Lp-spaces and their dense subspaces, Complex measures; Absolute continuity; Radon-Nykodym theorem.Product measures; Fubini's theorem; Convolutions.

Derivatives of measures; Functions of bounded variation; Fourier transform.

References :

Books:

W. Rudin: Real and Complex Analysis, Third edition, McGraw-Hill, International Editions, 1987.

### MA6120 ADVANCED COMPLEX ANALYSIS

Analytic Continuation: Direct Analytic Continuation, Monodromy Theorem, Poisson Integral Formula, Analytic Continuation via Reflection,

Representations for Meromorphic and Entire Functions: Infinite Sums and Meromorphic functions, Infinite Product of Complex Numbers, Infinite Products of Analytic Functions, Factorization of Entire Functions, The Gamma Function, The Zeta Function Jensen's Formula, The Order and the Genus of Entire Functions.

Mapping Theorems: Open Mapping Theorem and Hurwitz' Theorem, Basic Results on Univalent Functions, Normal Families, The Riemann Mapping Theorem, Bieberbach Conjecture, The Bloch-Landau Theorems Picard's Theorem.

References :

Books:

L. Ahlfors: Complex Analysis, 2nd ed., McGraw-Hill,New York, 1966.
 T.W. Gamelin: Complex Analysis, Springer-Verlag, 2001.
 S. Ponnusamy, S.: Foundations of Complex Analysis, 2nd ed., Narosa Publishing House, 508 pp., India (2005)
 S. Ponnusamy and H. Silverman, Complex Variables with Applications, 2006, 524 pp, Birkhaeuser, Boston.

# MA6140 FIXED POINT THEORY AND APPLICATIONS

Contraction Principle, and its variants and applications; Fixed points of nonexpansive maps and set valued maps, Brouwer -Schauder fixed point theorems, Ky Fan Best Approximation Theorem, Principle and Applications of KKM -maps, their variants and applications. Fixed Point Theorems in partially ordered spaces and other abstract spaces. Application of fixed point theory to Game theory and Mathematical Economics.

## References :

Books:

1. M.A.Khamsi and W.A.Kirk, An introduction to Metric Spaces and Fixed Point Theory, Wiley - Inter Sci. (2001)

 2. Sankatha Singh, Bruce Watson and Pramila Srivastava, Fixed Point Theory and Best Approximation: The KKM - map Principle, Kluwer Academic Publishers, 1997.
 3. Kim C. Border, Fixed point theorems with applications to economics and game theory,

Cambridge University Press, 1985.

# MA6150 BASIC OPERATOR THEORY

Dual space considerations: Representation of duals of the spaces  $c_{00}$  with p-norms,  $c_0$  and c with supremum-norm, l-p, C[a, b] and L^p. Reflexivity; Weak and weak\* convergences. Operators on Banach and Hilbert spaces: Compact operators between normed linear spaces; Integral operators as compact operators; Adjoint of operators between Hilbert spaces; Self-

adjoint, normal and unitary operators; Numerical range and numerical radius; Hilbert--Schmidt operators.

Spectral results for Banach and Hilbert space operators: Eigen spectrum, approximate eigen spectrum; Spectrum and resolvent; Spectral radius formula; Spectral mapping theorem; Riesz-Schauder theory; Spectral results for normal, self-adjoint and unitary operators; Functions of self-adjoint operators.

Spectral representation of operators: Spectral theorem and singular value representation for compact self-adjoint operators; Spectral theorem for self-adjoint operators.

### References :

Books:

1. T. Nair: Functional Analysis: A First Course, Wiley Eastern, 1981.

2. B.V. Limaye: Functional Analysis, Second Edition, New Age International, 1996.

## MA6180 INTRODUCTION TO ALGEBRAIC GEOMETRY

AFFINE AND PROJECTIVE VARIETIES: Noetherian rings and modules; Emmy Noether's theorem and Hilbert's Basissatz; Hilbert's Nullstellensatz; Affine and Projective algebraic sets; Krull's Hauptidealsatz; topological irreducibility, Noetherian decomposition; local ring, function field, transcendence degree and dimension theory; Quasi-Compactness and Hausdorffness; Prime and maximal spectra; Example: linear varieties, hypersurfaces, curves.

MORPHISMS: Morphisms in the category of commutative algebras over a commutative ring; behaviour under localization; morphisms of local rings; tensor products; Product varieties; standard embeddings like the segre- and the d-uple embedding.

RATIONAL MAPS: Relevance to function fields and birational classification; Example: classification of curves; blowing-up.

NONSINGULAR VARIETIES: Nonsingularity; Jacobian Criterion; singular locus; Regular local rings; Normal rings; normal varieties; Normalization; concept of desingularisation and its relevance to Classification Problems; Jacobian Conjecture; relationships between a ring and its completion; nonsingular curves.

INTERSECTIONS IN PROJECTIVE SPACE: Notions of multiplicity and intersection with examples.

References :

Books:

1. Robin Hartshorne, Algebraic Geometry, Graduate Texts in Mathematics (GTM) Vol.52, Corr. 8th Printing, 1997, Springer-Verlag.

2. C. Musili, Algebraic Geometry for Beginners, Texts and Readings in Mathematics 20, Hindustan Book Agency, India, 2001

## MA6190 Mathematical Logic

Basics of First order logic: Syntax of FL, formal semantics of FL, satisfiability, validity, equivalence, equality sentences, prenex form, Skolemization, Herbrand interpretation, Skolem-

Lowenheim theorem.

Proofs in FL: A Hilbert style axiom system, soundness and completeness, compactness, analytic tableax, mathematical theories, axiomatization of arithmetic.

Issues about FL: Undecidability, definability, expressibility, negation incompleteness, unprovability of consistency of arithmatic.

References :

Texts books:

 A. Singh and C. Goswami, Fundamentals of Logic, ICPR, New Delhi, 1998.
 S. Bilaniuk, A Problem Course in Mathematical Logic, GNU Free Documentation, http://euclid.trentu.ca/math/sb/pcml/, 2003.
 References:

 A. Singh, Logics for Computer Science, PHI Learning, New Delhi, 2003.
 R.M. Smullyan, A Beginner's guide to Mathematical Logic, Dover Publications, Inc. New York, 2014.
 H.B. Enderton, A Mathematical Introduction to Logic, 2nd Ed., Harcourt/Academic Press, New York, 2001.
 Yu.I. Manin, A Course in Mathematical Logic for Mathematicians, 2nd Ed., Springer-Verlag, New York, 2008.

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# **MA6200 THEORY OF COMPUTATION**

Grammars and Languages: Language basics, Regular expressions, Regular grammars, Contextfree grammars, context-sensitive grammars, unrestricted grammars, Chomsky hierarchy. Automata: Finite automata, pushdown automata, Pumping Lemmas and Closure properties, Turing machines and recursively enumerable languages.

Computability: Computable functions, non-recursively enumerable languages, Undecidability, Rice's theorem, Post's correspondence problem, Undecidability of validity problem of First Order Logic.

Complexity: Asymptotic order symbol, Space and Time complexity, Classes P and NP, NP-completeness, Cook-Levin tehorem, Other NP-complete problems.

## References :

Books:

1. K.Krithivasan and R.Rama, Introduction to Formal Languages, Automata and Computation, Pearson Education, 2009.

2. A.Singh, Elements of Computation Theory, Springer (In: Texts in Computer Science Series),2009.

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## MA6210 Combinatorial Optimization

Complexity of optimization problems: Complexity classes P and NP; Karp reduction; decision, solution and evaluation vertions of an optimization problem.

Design techniques for approximation algorithms: Greedy methods for knapsac, independent set

and TSP; sequential algorithms for scheduling, bin packing and graph coloring; local search algorithms for max-cut and TSP.

Approximation classes: Approximate solutions with guaranteed absolute error and relative error; approximability and non-approximability of TSP; limits to approximability (gap technique); complexity classes PTAS and FPTAS; strong NP-completeness and pseudo-polynomiality. Approximation algorithms for various problems: Set-cover, graph coloring, minimum multi-cut, edge coloring, bin packing.

References :

Texts books:

 G. Ausiello, P. Crescenzi, G. Gambosi, V. Kann, A. M. Protasi, Complexity and Approximation: Combinatorial Optimization Problems and Their Approximability Properties, Springer, 1999.
 References:

 V. Vazirani, Approximation Algorithms, Springer 2005.
 C. H. Papadimitriou, K. Steiglitz, Combinatorial optimization: Algorithms and Complexity, PHI 2001.
 J. Kleinberg, E. Tardos, Algorithm Design, Pearson India, 2007.

## MA6270 NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Parabolic Equations: explicit and implicit finite difference approximations to one - dimensional heat equation, Alternating Direction Implicit (ADI) method, Hyperbolic Equation: Characteristic method, finite difference solution of second order wave equation, Elliptic equations: finite difference method in polar coordinates, techniques near curved boundaries, improvement of accuracy, methods to accelerate the convergence, Convergence, consistency and stability analysis, Finite element method: types of integral formulations, one and two dimensional elements, Galerkin formulation, application to Dirichlet and Neumann problems.

#### References :

Books:

1. G D Smith, Numerical solution of partial differential equations: Finite difference methods, Oxford University press, 1977.

2. G. Evans, J. Blackledge, P. Yardley, Numerical Methods for Partial Differential Equations, 2nd edition, Springer, 2001.

## MA6312 MATHEMATICAL THEORY OF GAMES

### PART I: STATIC GAMES

1. Two-person zero-sum finite games: MAtrix games, Pure and mixed strategies, saddle point equilibrium strategies, extensive form games.

2. N-person finite games: Nash equilibrium, Refinements of Nash equilibrium, N-person games in extensive form.

3. Infinite games: Equilibrium strategies, continuous-kernel games, stackelberg equilibirum.

### PART II: DYNAMIC GAMES

3. Formulation of dynamic games: Discrete-time dynamic games, continuous-time dynamic games, mixed and behaviour strategies in dynamic games, representations of strategies along trajectories, time-consistency of optimal policies.

5. Equilibria of dynamic games: Open-loop and feedback equilibria for dynamic games in discrete-time, informational properties of Nash equilibrium in discrete-time dynamic games, Open-loop and Feedback equilibria for differential games.

6. Pursuit-evasion games: Necessary and sufficient conditions for equilibrium, captuability, singular surfaces.

References :

### TEXT BOOKS:

T.Basar and G.J.Olsder, Dynamic Noncooperative Game Theory, SIAM, 2nd edn, 1999 REFERENCES:

1. R. B. Myerson, Game Theory: Analysis of Conflict, Harvard University Press, 1999

2. M. J. Osborne and A.Rubinstein, A course in Game Theory, MIT Press, 1994

3. D. Fudenberg and J. Tirole, Game Theory, MIT Press, 1991

# MA6360 OPTIMIZATION TECHNIQUES

Hilbert spaces: The projection theorem - Approximation - Minimum norm problems. Calculus of variations: Extrema functionals - Gateax variatin - Euler-Lagrange equations - Problems with variable end points - Isoperimetric problem - Direct methods for solving variational problems. Optimal Control Theory: Pontryagin's maximum principle - Time optimal problems.

References :

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# MA6420 Algebraic Theory of Codes and Automata

Automata and words: Descriptional complexity of finite state automaton, local languages, syntactic semigroups, tree automata, words, Fine-Wilf theorem.

Codes: Basics of coding, types of codes, test for codes, flower automaton, finite transducers for codes.

Shift spaces: Finite and infinite, sofic shifts, entropy, Perron-Frobenius theory, finite state codes, cellular automata, dynamical systems.

References :

Texts books:

 J. Berstel, D. Perrin, C. Reutenauer, Codes and Automata, Cambridge University Press, 2010.
 D. Lind, B. Marcus, An introduction to Symbolic Dynamics and Coding, Cambridge University Press, 1995.

References:

1. M. Ito, Algebraic theory of Automata and Formal Languages, Cambridge University Press, 2004.

2. M. Lothaire, Combinatorics on Words, Cambridge University Press, 1997.

3. A. Salomaa, G.Rozenberg, Handbook of Formal Languages (Vol I), Springer, 1997.

4. J-P. Allouche, J.Shallit, Automatic Sequences, Cambridge University Press, 2003.

5. S. Ginsberg, Algebraic and Automata-Theoretic Properties of Formal Languages, North Holland Publishing Co., 1975.

6. R. Tao, Finite Automata and Applications to Cryptography, Springer, 2009.

# MA6460 COMPUTATIONAL FLUID DYNAMICS

Review of the governing equations of Incompressible viscous flows, Stream function -vorticity approach, upwind schemes, Primitive variables, Staggered grid, Artificial compressibility, pressure correction and vortex methods; Compressible inviscid flows, central schemes with combined and independent space time discretisation, Compressible viscous flows, Explicit, implicit and PISO methods; Grid generation: Structured and unstructured grid generation methods; Finite volume method to convection-diffusion equations.

References :

Books:

 P Wessling, Principles of Computational Fluid Dynamics, Springer, 1991.
 John D Anderson, Jr., Computational Fluid Dynamics, The Basics with Applications, McGraw-Hill, 1995.

# MA6470 COMMUTATIVE ALGEBRA

Rings and Modules, Localisation of Rings and Modules, Noetherian Rings and Modules, Primary Decomposition, Artinian rings, Integral Extensions, Going-up, Lying-over and Goingdown Theorems, Hilbert's Nullstellensatz, Noether's Normalisation, Dimension Theorem, Krull's Principal Ideal Theorem, Dedekind Domains.

References :

Books:1.M. F. Atiyah and I. G. Macdonald: Introduction to Commutative Algebra, Addision-Wesley , 1969.2.N. S. Gopalakrishnan: Commutative Algebra, Oxonian Press, 1984.

# MA6480 GALOIS THEORY

Algebraic Extensions of Fields, Splitting Fields and Algebraic Closures, Separable and Inseparable Extensions, Cyclotomic Polynomails and Extensions, Basics of Galois theory, Fundamental Theorem and Galois Theory, Finite Fields, Composite Extensions and simple Extensions, Cyclotomic Extensions and Abelian Extensions over q, Galois Group of Polynomials, Solvable and Radical extensions, Insolvability of the quintic equations.

### References :

Books:

D.S. Dummit and R.M Foote: Abstract Algebra, 2nd Edition, John-Wiley, 1999.
 S. Lang: Algebra. Revised third edition. Graudate Texts in Mathematics, 211. Springer-Verlag, New York, 2002.

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### **MA7015 Introduction to Cryptology**

Introduction: Brief introduction to number theory, Euclidean algorithm, Euler's totient function, Fermat's theorem and Euler's generalization, Chinese Remainder Theorem, primitive roots and discrete logarithms, Quadratic residues, Legendre and Jacobi symbols.

Private key cryptography: Stream ciphers, Block ciphers, DES and differential and linear cryptanalysis, Advanced encryption standards, Collision resistant hashing, Authenticated encryption: security against active attacks.

RSA public key cryptosystems: RSA system, primality testing, survey of factoring algorithms. Other public key cryptosystems: El Gamal public key cryptosystem, algorithms for discrete log problem.

References :

Texts books:

1. Introduction to Modern Cryptography by J. Katz and Y. Lindell.

References:

1. Cryptography: Theory and Practice by D. Stinson.

2. Handbook of Applied Cryptography by A. Menezes, P. C. Van Oorschot and S. A. Vanstone.

3. A Course in Number Theory and Cryptography by N. Koblitz.

### **MA7654 Algebraic Combinatorics**

Objectives: The course aims to reveal the fascinating interplay between Algebra, Combinatorics, and Graphs. It not only builds the fundamentals for students who plan to do PhD in Algebra/ Combinatorics / Graph theory and related topics, but also useful to students and researchers in other areas of science and engineering to which the methods of Algebra, Combinatorics, Graph theory may be applied.

## **Course Contents:**

Eigenvalues and Walks on graphs. Radon transform and hypercubes. Sperner property, lattices and boolean algebra. Enumeration under group actions. Ferrer's diagram, Young tableaux and Matrix Tree theorem. Applications to Electrical networks, planar graphs. Introduction to combinatorial commutative algebra. *References :* 

# **Text Books:**

Richard P Stanley, Algebraic Combinatorics : Walks - Trees - Tableaux and More, Springer, 2013.

A preliminary copy of the book is available from Stanley's webpage.

# **Reference Books:**

- 1. Richard P Stanley, Enumerative Combinatorics Volume 1, Springer. 2001
- 2. Richard P Stanley, Enumerative Combinatorics Volume 2, Springer, 2001.
- 3. Combinatorial Commutative Algebra. Erza Miller and Bernd Sturmfels. Springer, 2005.
- 4. Rafael H Villarreal, Monomial Algebras, CRC Press, 2015.
- 5. R B Bapat, Graphs and Matrices, Springer, 2014.

## MA5314 Differential geometry of manifolds

Revision of multivariate calculus , inverse and implicit function theorems Smooth manifolds, manifolds with boundary : definitions ,examples, smooth maps, diffeomorphisms, tangent vector, tangent plane, derivative (differential) of a smooth map immersions, submersions, embeddings and submanifolds orienation on manifolds vector fields and Lie bracket, Lie derivative Riemmanian metric, Riemmanian manifold, definition, example, pullback of a metric, vector bundles, tangent and cotangent bundle, Lie groups definition and examples, left , right and biinvariant Riemmanian metric, connections on manifolds, Riemmanian (Levi-Civita) connections, covariant derivative, connections on vector bundles geodesics, geodesic flow, exponential maps, map, minimizing properties of geodesics curvature, sectional curvature, scalar curvature, tensors on Riemmanian manifolds

References :

TextBooks: Manifolds and Differential geometry by J.M. Lee, publisher AMS,Roode Island,2009

ReferenceBooks:

- 1) Riemmanin geometry by Manfredo Peridiago Do Carmo, Publisher Birkhauser, 1992
- 2) Foundations of Differentiable manifolds and Lie groups F.K.Warner, publisher Springer, 1983.

Prerequisite: MA5310, MA5370, MA5380

## MA5315 Differential Topology

### **Description:**

To introduce manifolds, differential (smooth) structures on manifolds and classifying smooth manifolds up to the notion of diffeomorphisms

### **CourseContent:**

Revision of multivariate calculus, inverse and implicit function theorems, Smooth manifolds, manifolds with boundary, : definitions, examples, smooth maps, diffeomorphisms, tangent plane, derivative of smooth maps, immersions, submersions, local immersion and submersion theorems,

embeddings and submanifolds, regular and critical points of a smooth maps, regular and critical values of smooth maps, Transversality, homotopy and stability of transversality, Sards theorem, Whitney embedding theorem (We need to define partition of unity and its existence) The degree modulo 2 of a mapping, Oriented manifolds, The Brouwer degree Vector fields and Euler number, Poincare-Hopf index thorem Differential forms, integration on manifolds, Stoke's theorem on manifolds with boundary and introduction to De-Rham cohomolgy

References :

TextBooks: Differential topology : Victor Guillemin and Alan Pollack, Publisher: Prentice-Hall 1974

ReferenceBooks:

Topology from Differentiable viewpoint : John W Milnor , Princeton University Press, 1997
 Differential Manifolds : Antoni A Kosinski Dover Publications, Inc, New York 1993

Prerequisite: MA5310, MA5370, MA5380

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### MA6490 Introduction to Algebraic Number Theory

Commutative Algebra (General Theory of Dedekind Domains): Localization, Integral Dependence, DVR and Dedekind Domains, Modules over Dedekind Domains, Decomposition of primes in extensions. Class Groups and Dirichlet's Unit Theorem: Class groups, Norms and Traces, Discriminant, Norms of Ideals, Computing ring of integer in Quadratic Fields and Cyclotomic Fields, Quadratic Reciprocity, Minkowski's Bound, Dirichlet's Unit Theorem. Zeta and L-functions and applications: L-functions, Riemann Zeta functions, Dedekind Zeta functions, Class number formula, Frobenius Density Theorem, Dirichlet L-functions, Dirichlet's theorem on arithmetic progression. Theory of p-adic numbers: Valuations, Non-Archimedean absolute values, Completion and local fields, p-adic integers, Hensel's Lemma and applications, Ostrowski's theorem.

### References :

**TextBooks :** Gerald J. Janusz: Algebraic Number Fields, Second Edition. Graduate Studies in Mathematics, Volume 7. (American Mathematical Society, 1996) **Reference Books:** Daniel A. Marcus, Number Fields,Springer, New York (1977). Universitext Serge Lang, Algebraic Number Theory, Second Edition. Graduate Texts in Mathematics, Volume 110 (Springer, New York, 1994) J. Neukirch, Algebraic Number Theory. Grundlehren Math. Wiss. (Springer, Berlin, 1999) J.W.S. Cassels and A. Frohlich, Algebraic Number Theory, Second Edition. London Mathematical Society, 2010.